14

SIMILARITY OF TRIANGLES

Looking around you will see many objects which are of the same shape but of same or different sizes. For examples, leaves of a tree have almost the same shape but same or different sizes. Similarly, photographs of different sizes developed from the same negative are of same shape but different sizes, the miniature model of a building and the building itself are of same shape but different sizes. **All those objects which have the same shape but not necessarily the same size are called similar objects.**

Let us examine the similarity of plane figures (Fig. 14.1):

(i) Two line-segments of the same length are congruent as well as similar and of different lengths are similar but not congruent.

Fig. 14.1 (i)

(ii) Two circles of the same radius are congruent as well as similar and circles of different radii are similar but not congruent.

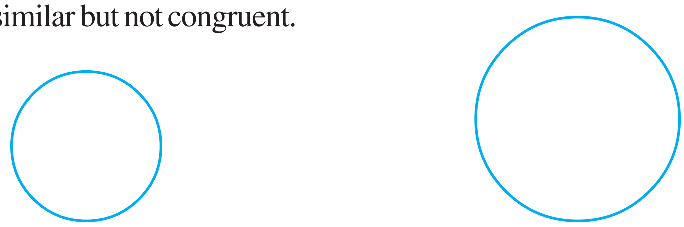


Fig. 14.1 (ii)

(iii) Two equilateral triangles of different sides are similar but not congruent.

Fig. 14.1 (iii)

(iv) Two squares of different sides are similar but not congruent.

Fig. 14.1 (iv)

In this lesson, we shall study about the concept of similarity, particularly similarity of triangles and the conditions thereof. We shall also study about various results related to them.

OBJECTIVES

After studying this lesson, you will be able to

- *identify similar figures;*
- *distinguish between congruent and similar plane figures;*
- *prove that if a line is drawn parallel to one side of a triangle then the other two sides are divided in the same ratio;*
- *state and use the criteria for similarity of triangles viz. AAA, SSS and SAS;*
- *verify and use unstarred results given in the curriculum based on similarity experimentally;*
- *prove the Baudhayan/Pythagoras Theorem;*
- *apply these results in verifying experimentally (or proving logically) problems based on similar triangles.*

EXPECTED BACKGROUND KNOWLEDGE

- knowledge of plane figures like triangles, quadrilaterals, circles, rectangles, squares, etc.
- criteria of congruency of triangles.
- finding squares and square-roots of numbers.
- ratio and proportion.
- Interior and exterior angles of a triangle.

14.1 SIMILAR PLANE FIGURES

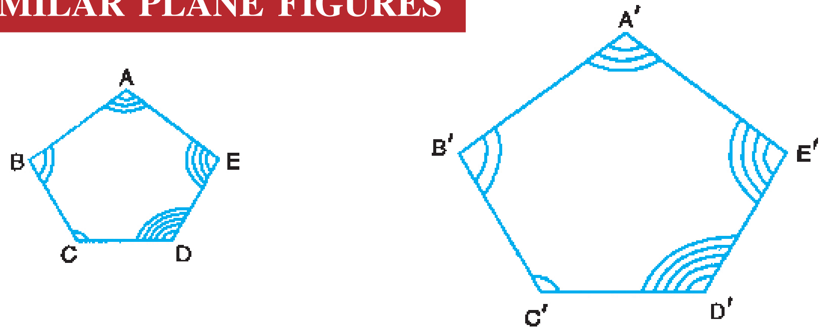


Fig. 14.2

In Fig. 14.2, the two pentagons seem to be of the same shape.

We can see that if $\angle A = \angle A'$, $\angle B = \angle B'$, $\angle C = \angle C'$, $\angle D = \angle D'$ and $\angle E = E'$ and $\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CD}{C'D'} = \frac{DE}{D'E'} = \frac{EA}{E'A'}$. then the two pentagons are similar. Thus we say that

Any two polygons, with corresponding angles equal and corresponding sides proportional, are similar.

Thus, two polygons are similar, if they satisfiy the following two conditions:

(i) Corresponding angles are equal.

(ii) The corresponding sides are proportional.

Even if one of the conditions does not hold, the polygons are not similar as in the case of a rectangle and square given in Fig. 14.3. Here all the corresponding angles are equal but the corresponding sides are not proportional.

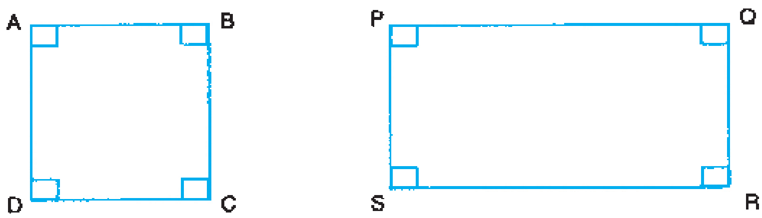


Fig. 14.3

14.2 BASIC PROPORTIONALITY THEORM

We state below the Basic Proportionality Theorm:

If a line is drawn parallel to one side of a triangle intersecting the other two sides, the other two sides of the triangle are divided proportionally.

Thus, in Fig. 14.4, DE || BC, According to the above result

$$\frac{AD}{DB} = \frac{AE}{EC}$$

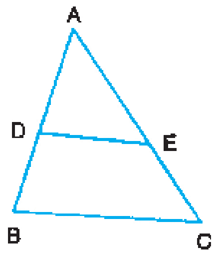


Fig. 14.4

We can easily verify this by measuring AD, DB, AE and EC. You will find that

$$\frac{AD}{DB} = \frac{AE}{EC}$$

We state the converse of the above result as follows:

If a line divides any two sides of a triangle in the same ratio, the line is parallel to third side of the triangle.

Thus, in Fig 14.4, if DE divides side AB and AC of ΔABC such that $\frac{AD}{DB} = \frac{AE}{EC}$, then DE || BC.

We can verify this by measuring ∠ADE and ∠ABC and finding that

$$\angle ADE = \angle ABC$$

These being corresponding angles, the line DE and BC are parallel.

We can verify the above two results by taking different triangles.

Let us solve some examples based on these.

Example 14.1: In Fig. 14.5, DE || BC. If AD = 3 cm, DB = 5 cm and AE = 6 cm, find AC.

Solution: DE || BC (Given). Let EC = x

$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$

$\therefore \quad \frac{3}{5} = \frac{6}{x}$

$\Rightarrow \quad 3x = 30$

$\Rightarrow \quad x = 10$

$\therefore \quad EC = 10$ cm

$\therefore \quad AC = AE + EC = 16$ cm

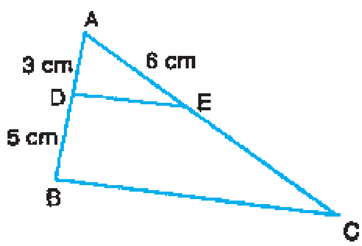


Fig. 14.5

Example 14.2: In Fig. 14.6, AD = 4 cm, DB = 5 cm, AE = 4.5 cm and EC = $5\frac{5}{8}$ cm. Is DE || BC? Given reasons for your answer.

Solution: We are given that AD = 4 cm and DB = 5 cm

$$\therefore \qquad \frac{AD}{DB} = \frac{4}{5}$$

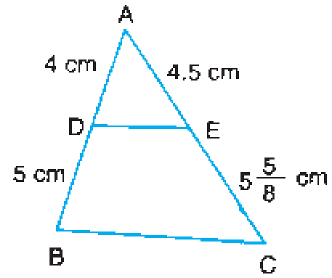


Fig. 14.6

Similarly, $$\frac{AE}{EC} = \frac{4.5}{\frac{45}{8}} = \frac{9}{2} \times \frac{8}{45} = \frac{4}{5}$$

$$\therefore \qquad \frac{AD}{DB} = \frac{AE}{EC}$$

$\therefore$ According to converse of Basic Proportionality Theorem

DE || BC

CHECK YOUR PROGRESS 14.1

1. In Fig. 14.7 (i) and (ii), PQ || BC. Find the value of x in each case.

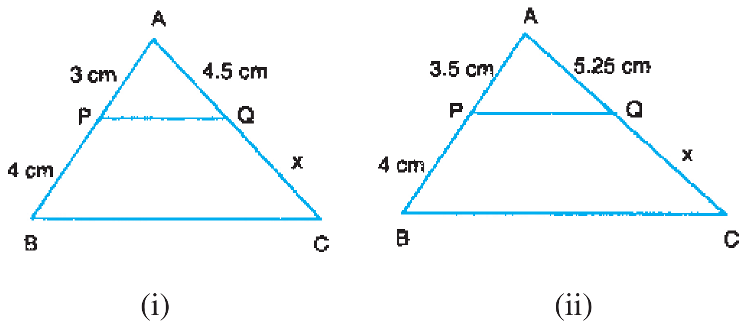
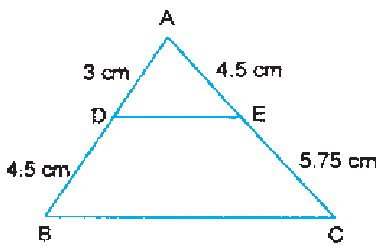


Fig. 14.7

2. In Fig. 14.8 [(i)], find whether DE || BC is parallel to BC or not? Give reasons for your answer.



(i)

Fig. 14.8

14.3 BISECTOR OF AN ANGLE OF A TRIANGLE

We now state an important result as given below:

The bisector of an interior angle of a triangle divides the opposite side in the ratio of sides containing the angle.

According to the above result, if AD is the internal bisector of $\angle$A of ΔABC, then

$$\frac{BD}{DC} = \frac{AB}{AC} \text{ (Fig. 14.9)}$$

We can easily verify this by measuring BD, DC, AB and AC and finding the ratios. We will find that

$$\frac{BD}{DC} = \frac{AB}{AC}$$

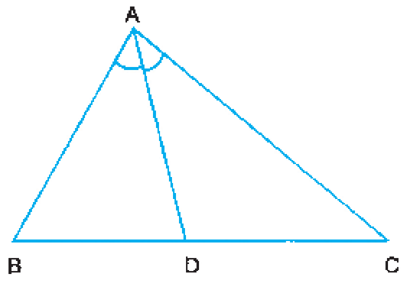


Fig. 14.9

Repeating the same activity with other triangles, we may verify the result.

Let us solve some examples to illustrate this.

Example 14.3: The sides AB and AC of a triangle are of length 6 cm and 8 cm respectively. The bisector AD of $\angle$A intersects the opposite side BC in D such that BD = 4.5 cm (Fig. 14.10). Find the length of segment CD.

Solution: According to the above result, we have

$$\frac{BD}{DC} = \frac{AB}{AC}$$

($\because$ AD is internal bisector of $\angle$A of ΔABC)

or $\quad \frac{4.5}{x} = \frac{6}{8}$

$\Rightarrow \quad 6x = 4.5 \times 8$

$x = 6$

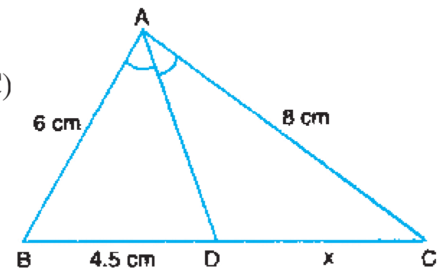


Fig. 14.10

i.e., the length of line-segment CD = 6 cm.

Example 14.4: The sides of a triangle are 28 cm, 36 cm and 48 cm. Find the lengths of the line-segments into which the smallest side is divided by the bisector of the angle opposite to it.

Solution: The smallest side is of length 28 cm and the sides forming $\angle$A opposite to it are 36 cm and 48 cm. Let the angle bisector AD meet BC in D (Fig. 14.11).

$$\therefore \qquad \frac{BD}{DC} = \frac{36}{48} = \frac{3}{4}$$

$$\Rightarrow \quad 4BD = 3DC \quad \text{or} \quad BD = \frac{3}{4}DC$$

$$BC = BD + DC = 28 \text{ cm}$$

$$\therefore \qquad DC + \frac{3}{4}DC = 28$$

$$\therefore \qquad DC = \left(28 \times \frac{4}{7}\right) \text{cm} = 16 \text{ cm}$$

$$\therefore \qquad BD = 12 \text{ cm and } DC = 16 \text{ cm}$$

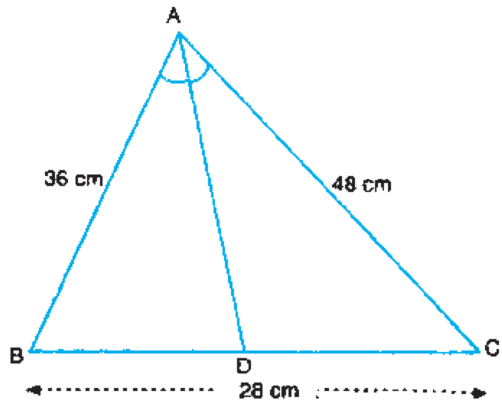


Fig. 14.11

CHECK YOUR PROGRESS 14.2

1. In Fig. 14.12, AD is the bisector of $\angle A$, meeting BC in D. If AB = 4.5 cm, BD = 3 cm, DC = 5 cm, find x.

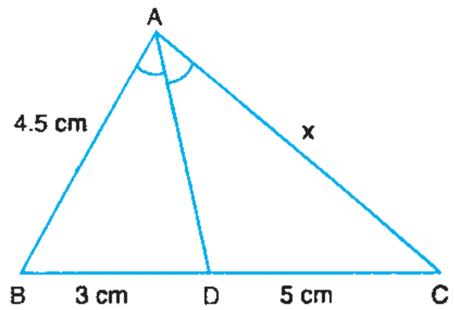


Fig. 14.12

2. In Fig. 14.13, PS is the bisector of $\angle P$ of ΔPQR. The dimensions of some of the sides are given in Fig. 14.13. Find x.

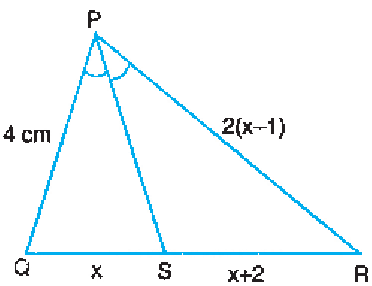


Fig. 14.13

3. In Fig. 14.14, RS is the bisector of ∠R of ΔPQR. For the given dimensions, express p, the length of QS in terms of x, y and z.

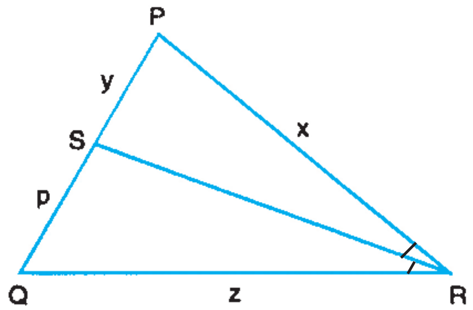


Fig. 14.14

14.4 SIMILARITY OF TRIANGLES

Triangles are special type of polygons and therefore the conditions of similarity of polygons also hold for triangles. Thus,

Two triangles are similar if

(i) their corresponding angles are equal, and

(ii) their corresponding sides are proportional.

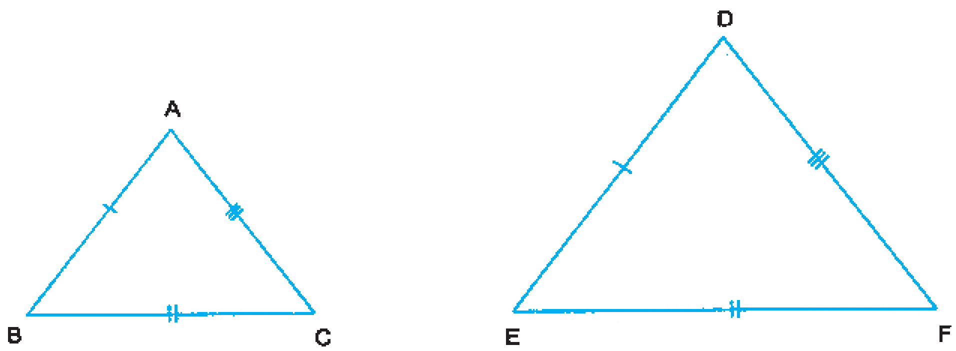


Fig. 14.15

We say that ΔABC is similar to ΔDEF and denote it by writing

ΔABC ~ ΔDEF (Fig. 14.15)

The symbol '~' stands for the phrase "is similar to"

If ΔABC ~ ΔDEF, then by definition

$\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$ and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$.

14.4.1 AAA Criterion for Similarity

We shall show that in the case of triangles if either of the above two conditions is satisfied then the other automatically holds.

Let us perform the following experiment.

Construct two Δ's ABC and PQR in which $\angle P = \angle A$, $\angle Q = \angle B$ and $\angle R = \angle C$ as shown in Fig. 14.16.

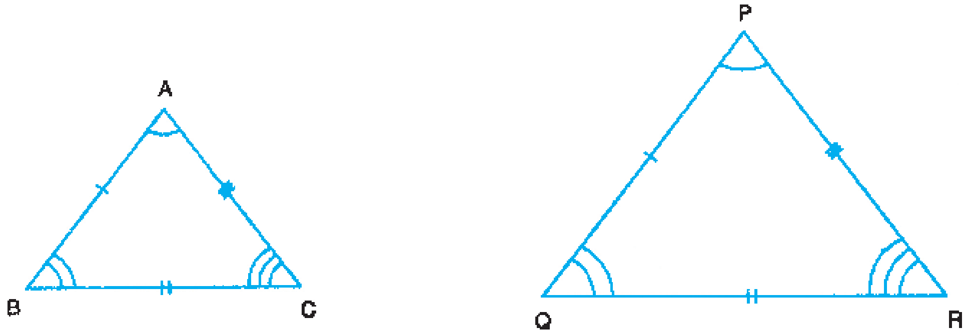


Fig. 14.16

Measure the sides AB, BC and CA of the ΔABC and also measure the sides PQ, QR and RP of ΔPQR.

Now find the ratio $\frac{AB}{PQ}, \frac{BC}{QR}$ and $\frac{CA}{RP}$.

What do you find? You will find that all the three ratios are equal and therefore the triangles are similar.

Try this with different triangles with equal corresponding angles. You will find the same result.

Thus, we can say that:

If in two triangles, the corresponding angles are equal the triangles are similar

This is called AAA similarity criterion.

14.4.2 SSS Criterion for Similarity

Let us now perform the following experiment:

Draw a triangle ABC with AB = 3 cm, BC = 4.5 cm and CA = 3.5 cm [Fig. 14.17 (i)].

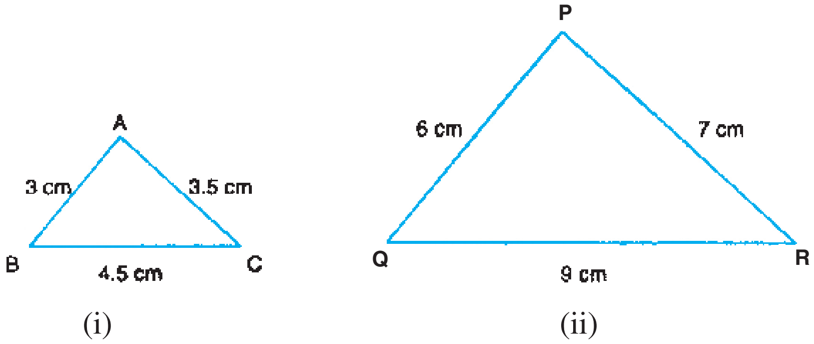


Fig. 14.17

Draw another ΔPQR as shown in Fig. 14.17(ii), with PQ = 6 cm, QR = 9 cm and PR = 7 cm.

We can see that $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

i.e., the sides of the two triangles are proportional.

Now measure ∠A, ∠B and ∠C of ΔABC and ∠P, ∠Q and ∠R of ΔPQR.

You will find that ∠A = ∠P, ∠B = ∠Q and ∠C = ∠R.

Repeat the experiment with another two triangles having corresponding sides proportional, you will find that the corresponding angles are equal and so the triangles are similar.

Thus, we can say that

If the corresponding sides of two triangles are proportional the triangles are similar.

14.4.3 SAS Criterian for Similarity

Let us conduct the following experiment.

Take a line AB = 3 cm and at A construct an angle of 60°. Cut off AC = 4.5 cm. Join BC.

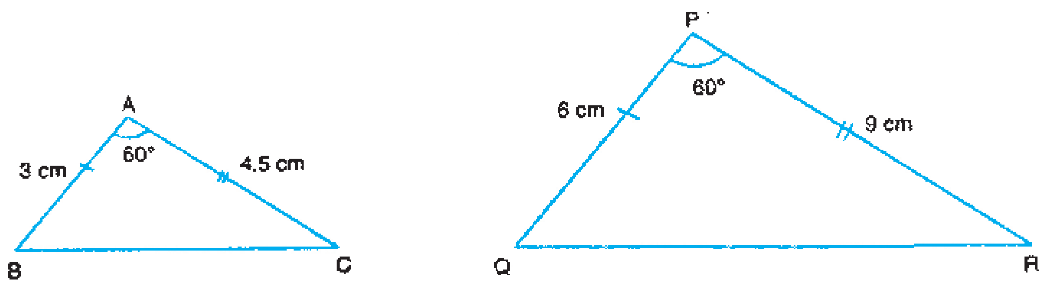


Fig. 14.18

Now take PQ = 6 cm. At P, draw an angle of 60° and cut off PR = 9 cm (Fig. 14.18) and join QR.

Measure $\angle B$, $\angle C$, $\angle Q$ and $\angle R$. We shall find that $\angle B = \angle Q$ and $\angle C = \angle R$

Thus, $\Delta ABC \sim \Delta PQR$

Thus, we conclude that

If one angle of a triangle is equal to one angle of the other triangle and the sides containing these angles are proportional, the triangles are similar.

Thus, we have three important criteria for the similarity of triangles. They are given below:

(i) **If in two triangles, the corresponding angles are equal, the triangles are similar.**

(ii) **If the corresponding sides of two triangles are proportional, the triangles are similar.**

(iii) **If one angle of a triangle is equal to one angle of the other triangle and the sides containing these angles are proportional, the triangles are similar.**

Example 14.5: In Fig. 14.19 two triangles ABC and PQR are given in which $\angle A = \angle P$ and $\angle B = \angle Q$. Is $\Delta ABC \sim \Delta PQR$?.

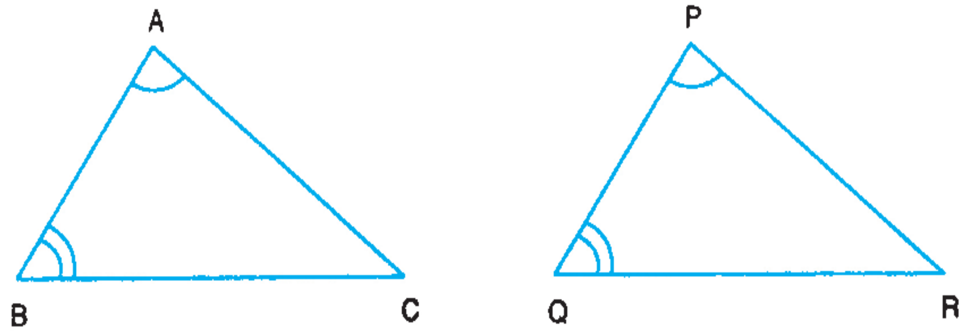


Fig. 14.19

Solution: We are given that

$$\angle A = \angle P \text{ and } \angle B = \angle Q$$

We also know that

$$\angle A + \angle B + \angle C = \angle P + \angle Q + \angle R = 180^\circ$$

Therefore $\angle C = \angle R$

Thus, according to first criterion of similarity (AAA)

$$\Delta ABC \sim \Delta PQR$$

Example 14.6: In Fig. 14.20, ΔABC ~ ΔPQR. If AC = 4.8 cm, AB = 4 cm and PQ = 9 cm, find PR.

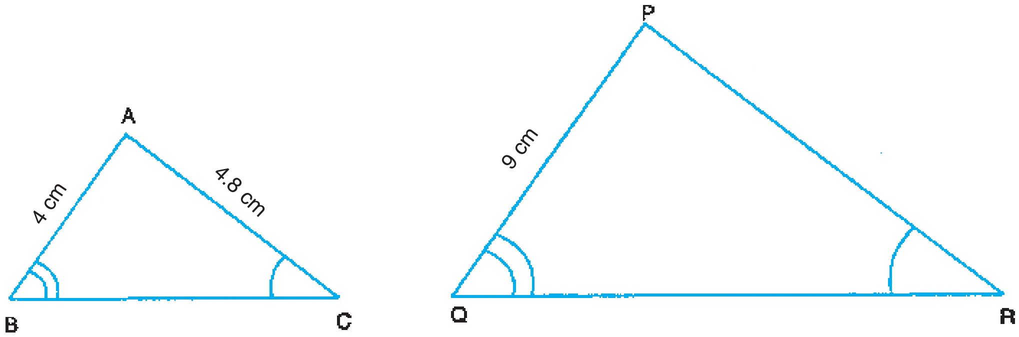


Fig. 14.20

Solution: It is given that ΔABC ~ ΔPQR

$\therefore \quad \frac{AB}{PQ} = \frac{AC}{PR}$

Let $\quad PR = x$ cm

$\therefore \quad \frac{4}{9} = \frac{4.8}{x}$

$\Rightarrow \quad 4x = 9 \times 4.8$

$\Rightarrow \quad x = 10.8$

i.e., $\quad PR = 10.8$ cm.

CHECK YOUR PROGRESS 14.3

Find values of x and y of ΔABC ~ ΔPQR in the following figures:

(i)

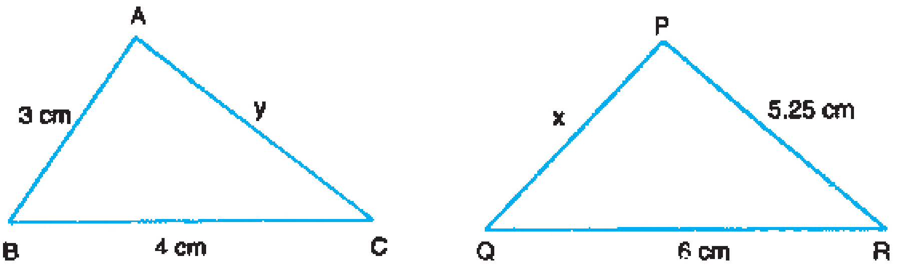


Fig. 14.21

(ii)

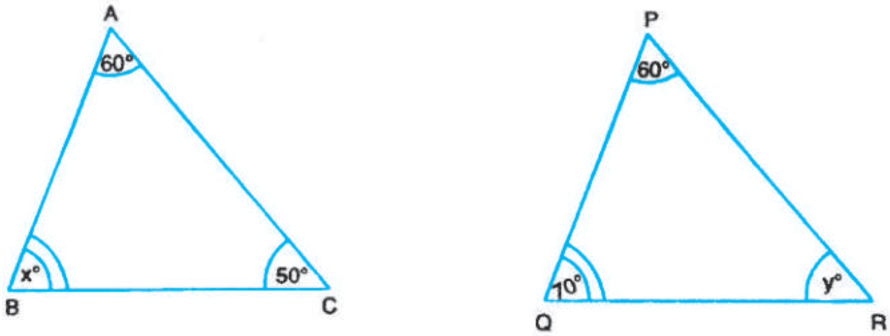


Fig. 14.22

(iii)

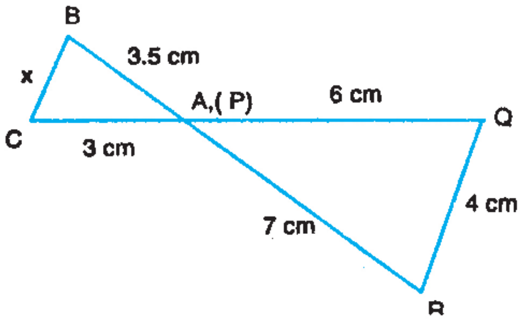


Fig. 14.23

14.5 SOME MORE IMPORTANT RESULTS

Let us study another important result on similarity in connection with a right triangle and the perpendicular from the vertex of right angle to the opposite side. We state the result below and try to verify the same.

If a perpendicualr is drawn from the vertex of the right angle of a right triangle to the hypotenuse, the triangles on each side of the perpendicular are similar to each other and to the original triangle.

Let us try to verify this by an activity.

Draw a ΔABC, right angled at A. Draw $AD \perp$ to the hypoenuse BC, meeting it in D.

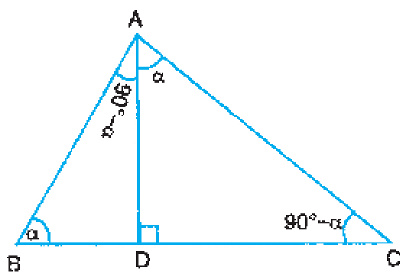


Fig. 14.24

Let $\angle DBA = \alpha$,

As $\angle ADB = 90^o$, $\angle BAD = 90^o - \alpha$

As $\angle BAC = 90^o$ and $\angle BAD = 90^o - \alpha$

Therefore $\angle DAC = \alpha$

Similarly $\angle DCA = 90^o - \alpha$

$\therefore$ ΔADB and ΔCDA are similar, as it has all the corresponding angles equal.

Also, the angles B, A and C of ΔBAC are α, 90^o and $90^o - \alpha$ respectively.

$$\therefore \quad \Delta ADB \sim \Delta CDA \sim \Delta CAB$$

Another important result is about relation between corresponding sides and areas of similar triangles.

It states that

The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

Let us verify this result by the following activity. Draw two right triangles ABC and PQR which are similar i.e., their sides are proportional (Fig. 14.25).

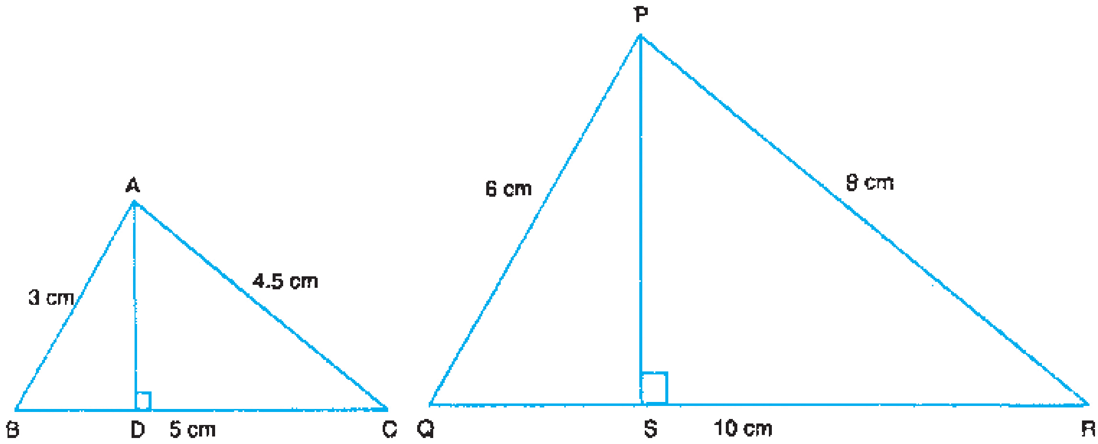


Fig. 14.25

Draw $AD \perp BC$ and $PS \perp QR$.

Measure the lengths of AD and PS.

Find the product $AD \times BC$ and $PS \times QR$

You will find that $AD \times BC = BC^2$ and $PS \times QR = QR^2$

Now $\quad AD \times BC = 2 \,.\, \text{Area of } \Delta ABC$

$\quad\quad PS \times QR = 2.\ \text{Area of } \Delta PQR$

$$\therefore \quad \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \frac{AD \times BC}{PS \times QR} = \frac{BC^2}{QR^2} \quad\quad ...(i)$$

$$\text{As} \quad \frac{BC}{QR} = \frac{AB}{PQ} = \frac{AC}{PR}$$

$$\therefore \quad \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \frac{BC^2}{QR^2} = \frac{AB^2}{PQ^2} = \frac{AC^2}{PR^2}$$

The activity may be repeated by taking different pairs of similar triangles.

Let us illustrate these results with the help of examples.

Example 14.7: Find the ratio of the area of two similar triangles if one pair of their corresponding sides are 2.5 cm and 5.0 cm.

Solution: Let the two triangles be ABC and PQR

Let $\quad BC = 2.5$ cm and $QR = 5.0$ cm

$$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta PQR)} = \frac{BC^2}{QR^2} = \frac{(2.5)^2}{(5.0)^2} = \frac{1}{4}$$

Example 14.8: In a ΔABC, $PQ \parallel BC$ and intersects AB and AC at P and Q respectively.

If $\frac{AP}{BP} = \frac{2}{3}$ find the ratio of areas ΔAPQ and ΔABC.

Solution: In Fig 14.26

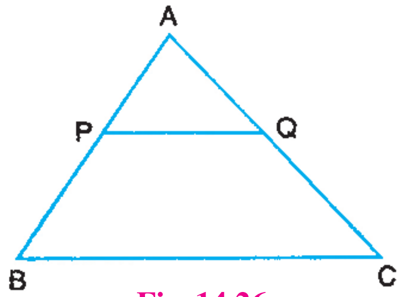


Fig. 14.26

$PQ \parallel BC$

$$\therefore \quad \frac{AP}{BP} = \frac{AQ}{QC} = \frac{2}{3}$$

$$\therefore \quad \frac{BP}{AP} = \frac{QC}{AQ} = \frac{3}{2}$$

$$\therefore 1 + \frac{BP}{AP} = 1 + \frac{QC}{AQ} = 1 + \frac{3}{2} = \frac{5}{2}$$

$$\Rightarrow \frac{AB}{AP} = \frac{AC}{AQ} = \frac{5}{2} \quad \Rightarrow \frac{AP}{AB} = \frac{AQ}{AC} = \frac{2}{5}$$

$$\therefore \quad \Delta APQ \sim \Delta ABC$$

$$\therefore \frac{\text{Area}(\Delta APQ)}{\text{Area}(\Delta ABC)} = \frac{AP^2}{AB^2} = \left(\frac{AP}{AB}\right)^2 = \left(\frac{2}{5}\right)^2 = \frac{4}{25} (\because \Delta APQ \sim \Delta ABC)$$

CHECK YOUR PROGRESS 14.4

1. In Fig. 14.27, ABC is a right triangle with $A = 90^\circ$ and $C = 30^\circ$. Show that $\Delta DAB \sim \Delta DCA \sim \Delta ACB$.

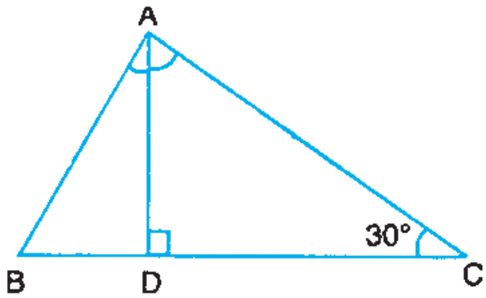


Fig. 14.27

2. Find the ratio of the areas of two similar triangles if two of their corresponding sides are of length 3 cm and 5 cm.

3. In Fig. 14.28, ABC is a triangle in which DE || BC. If AB = 6 cm and AD = 2 cm, find the ratio of the areas of ΔADC and trapezium DBCE.

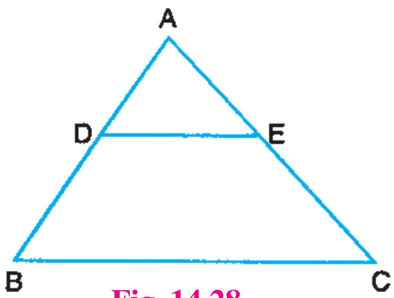


Fig. 14.28

4. P, Q and R are respectively the mid-points of the sides AB, BC and CA of the ΔABC. Show that the area of ΔPQR is one-fourth the area of ΔABC.

5. In two similar triangles ABC and PQR, if the corresponding altitudes AD and PS are in the ratio of 4 : 9, find the ratio of the areas of ΔABC and ΔPQR.

$$\left[\text{Hint : Use}\frac{AB}{PQ}=\frac{AD}{PS}=\frac{BC}{QR}=\frac{CA}{PR}\right]$$

6. If the ratio of the areas of two similar triangles is 16 : 25, find the ratio of their corresponding sides.

14.6 BAUDHYAN/PYTHAGORAS THEOREM

We now prove an important theorem, called Baudhayan/Phythagoras Theorem using the concept of similarity.

Theorem: In a right triangle, the square on the hypotenuse is equal to sum of the squares on the other two sides.

Given: A right triangle ABC, in which $\angle B = 90^o$.

To Prove: $AC^2 = AB^2 + BC^2$

Construction: From B, draw $BD \perp AC$ (See Fig. 14.29)

Proof: $BD \perp AC$

$\therefore \quad \Delta ADB \sim \Delta ABC \qquad ...(i)$

and $\quad \Delta BDC \sim \Delta ABC \qquad ...(ii)$

From (i), we get $\dfrac{AB}{AC} = \dfrac{AD}{AB}$

$\Rightarrow \quad AB^2 = AC \cdot AD \qquad ...(X)$

From (ii), we get $\dfrac{BC}{AC} = \dfrac{DC}{BC}$

$\Rightarrow \quad BC^2 = AC \cdot DC \qquad ...(Y)$

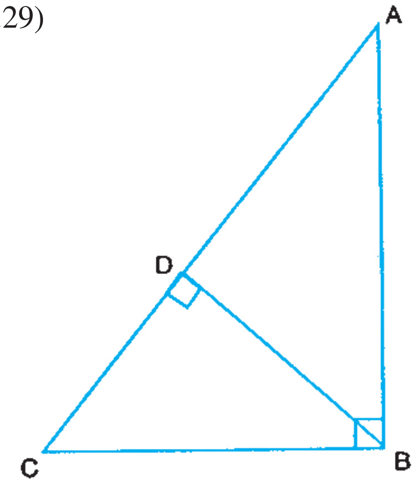


Fig. 14.29

Adding (X) and (Y), we get

$$AB^2 + BC^2 = AC\,(AD + DC)$$

$$= AC \cdot AC = AC^2$$

The theorem is known after the name of famous Greek Mathematician Pythagoras. This was originally stated by the Indian mathematician Baudhayan about 200 years before Pythagoras in about 800 BC.

14.6.1 Converse of Pythagoras Theorem

The conserve of the above theorem states:

In a triangle, if the square on one side is equal to sum of the squares on the other two sides, the angle opposite to first side is a right angle.

This result can be verified by the following activity.

Draw a triangle ABC with side 3 cm, 4 cm and 5 cm.

i.e., $\quad AB = 3$ cm, $BC = 4$ cm

and $AC = 5$ cm (Fig. 14.30)

You can see that $AB^2 + BC^2 = (3)^2 + (4)^2$

$$= 9 + 16 = 25$$

$$AC^2 = (5)^2 = 25$$

$$\therefore AB^2 + BC^2 = AC^2$$

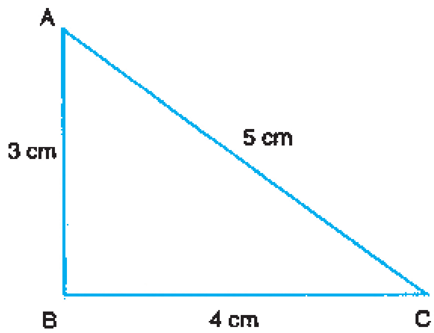


Fig. 14.30

The triangle in Fig. 14.30 satisfies the condition of the above result.

Measure $\angle ABC$, you will find that $\angle ABC = 90^o$. Construct triangles of sides 5 cm, 12 cm and 13 cm, and of sides 7 cm, 24 cm, 25 cm. You will again find that the angles opposite to side of length 13 cm and 25 cm are 90^o in each case.

Example 14.9: In a right triangle, the sides containing the right angle are of length 5 cm and 12 cm. Find the length of the hypotenuse.

Solution: Let ABC be the right triangle, right angled at B.

$\therefore$ AB = 5 cm, BC = 12 cm

Also, $AC^2 = BC^2 + AB^2$

$= (12)^2 + (5)^2$

$= 144 + 125$

$= 169$

$\therefore$ AC $= 13$

i.e., the length of the hypotenuse is 13 cm.

Example 14.10: Find the length of diagonal of a rectangle the lengths of whose sides are 3 cm and 4 cm.

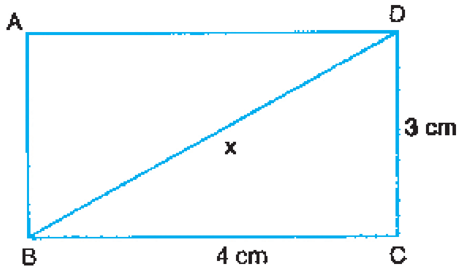


Fig. 14.31

Solution: In Fig. 14.31, is a rectangle ABCD. Join the diagonal BD. Now DCB is a right triangle.

$\therefore$ $BD^2 = BC^2 + CD^2$

$= 4^2 + 3^2$

$= 16 + 9 = 25$

BD $= 5$

i.e., the length of diagonal of rectangle ABCD is 5 cm.

Example 14.11: In an equilateral triangle, verify that three times the square on one side is equal to four times the square on its altitude.

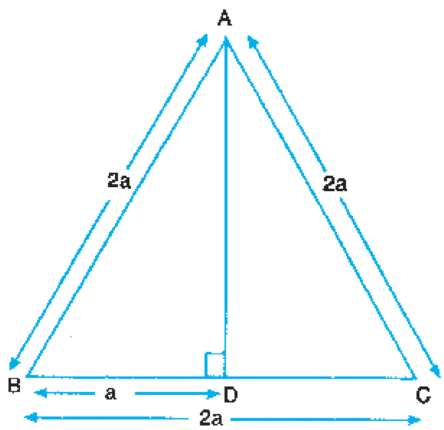


Fig. 14.32

Solution: The altitude $AD \perp BC$

and BD = CD (Fig. 14.32)

Let AB = BC = CA = 2a

and BD = CD = a

Let AD = x

$\therefore$ $x^2 = (2a)^2 - (a)^2 = 3a^2$

$3. (\text{Side})^2 = 3. (2a)^2 = 12\ a^2$

$4. (\text{Altitude})^2 = 4.\ 3a^2 = 12a^2$

Hence the result.

Example 14.12: ABC is a right triangle, right angled at C. If CD, the length of perpendicular from C on AB is p, BC = a, AC = b and AB = c (Fig. 14.33), show that:

(i) pc = ab

(ii) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

Solution: (i) $CD \perp AB$

$\therefore \Delta ABC \sim \Delta ACD$

$\therefore \frac{c}{b} = \frac{a}{p}$

$\Rightarrow pc = ab$

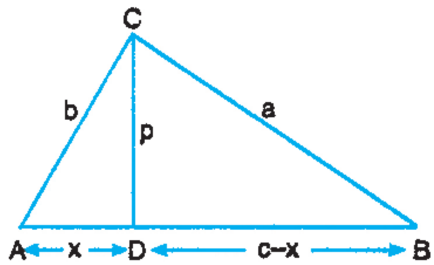


Fig. 14.33

(ii) $AB^2 = AC^2 + BC^2$

or $c^2 = b^2 + a^2$

$$\left(\frac{ab}{p}\right)^2 = b^2 + a^2$$

or $$\frac{1}{p^2} = \frac{a^2+b^2}{a^2b^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

CHECK YOUR PROGRESS 14.5

1. The sides of certain triangles are given below. Determine which of them are right triangles: [AB = c, BC = a, CA = b]

   (i) a = 4 cm, b = 5 cm, c = 3 cm

   (ii) a = 1.6 cm, b = 3.8 cm, c = 4 cm

   (iii) a = 9 cm, b = 16 cm, c = 18 cm

   (iv) a = 7 cm, b = 24 cm, c = 25 cm

2. Two poles of height 6 m and 11 m, stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.

3. Find the length of the diagonal of a square of side 10 cm.

4. In Fig. 14.34, $\angle C$ is acute and $AD \perp BC$. Show that $AB^2 = AC^2 + BC^2 - 2\,BC.\,DC$.

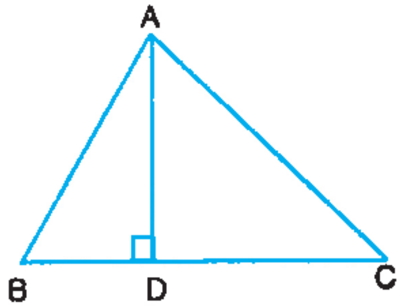


Fig. 14.34

5. L and M are the mid-points of the sides AB and AC of ΔABC, right angled at B. Show that $4LC^2 = AB^2 + 4\,BC^2$

6. P and Q are points on the sides CA and CB respectively of ΔABC, right angled at C Prove that $AQ^2 + BP^2 = AB^2 + PQ^2$

7. PQR is an isosceles right triangle with $\angle Q = 90^o$. Prove that $PR^2 = 2PQ^2$.

8. A ladder is placed against a wall such that its top reaches upto a height of 4 m of the wall. If the foot of the ladder is 3 m away from the wall, find the length of the ladder.

LET US SUM UP

- Objects which have the same shape but different or same sizes are called similar objects.
- Any two polygons, with corresponding angles equal and corresponding sides proportional are similar.
- If a line is drawn parallel to one-side of a triangle, it divides the other two sides in the same ratio and its converse.
- The bisector of an interior angle of a triangle divides the opposite side in the ratio of sides containing the angle.
- Two triangles are said to be similar, if

  (a) their corresponding angles are equal **and**

  (b) their corresponding sides are proportional
- Criteria of similarity

  - AAA criterion

  - SSS criterion

  - SAS criterion

- If a perpendicular is drawn from the vertex of the right angle of a right angled triangle to the hypotenuse, the triangles so formed are similar to each other and to the given triangle.
- The ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding sides.
- In a right triangle, the square on the hypotenuse is equal to sum of the squares on the remaining two sides – (Baudhayan Pythagoras Theorem).
- In a triangle, if the square on one side is equal to the sum of the squares on the remaining two sides, then the angle opposite to the first side is a right angle – converse of (Baudhayan) Pythagoras Theorem.

TERMINAL EXERCISE

1. Write the criteria for the similarity of two polygons.
2. Enumerate different criteria for the similarity of the two triangles.
3. In which of the following cases, Δ's ABC and PQR are similar.

   (i) $\angle A = 40^o$, $\angle B = 60^o$, $\angle C = 80^o$, $\angle P = 40^o$, $\angle Q = 60^o$ and $\angle R = 80^o$

   (ii) $\angle A = 50^o$, $\angle B = 70^o$, $\angle C = 60^o$, $\angle P = 50^o$, $\angle Q = 60^o$ and $\angle R = 70^o$

   (iii) AB = 2.5 cm, BC = 4.5 cm, CA = 3.5 cm

   PQ = 5.0 cm, QR = 9.0 cm, RP = 7.0 cm

   (iv) AB = 3 cm, QR = 7.5 cm, RP = 5.0 cm

   PQ = 4.5 cm, QR = 7.5 cm, RP = 6.0 cm.
4. In Fig. 14.35, AD = 3 cm, AE = 4.5 cm, DB = 4.0 cm, find CE, give that DE || BC.

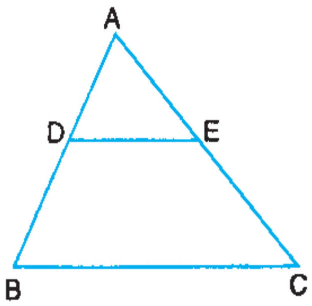


Fig. 14.35

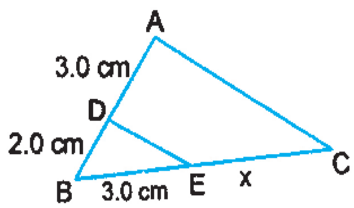


Fig. 14.36

5. In Fig. 14.36, DE || AC. From the dimensions given in the figure, find the value of x.

6. In Fig. 14.37 is shown a ΔABC in which AD = 5 cm, DB = 3 cm, AE = 2.50 cm and EC = 1.5 cm. Is DE || BC? Give reasons for your answer.

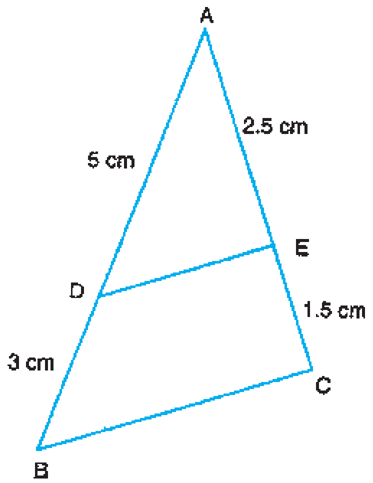


Fig. 14.37

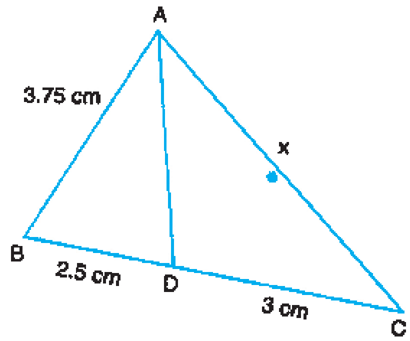


Fig. 14.38

7. In Fig. 14.38, AD is the internal bisector of $\angle A$ of ΔABC. From the given dimensions, find x.

8. The perimeter of two similar triangles ABC and DEF are 12 cm and 18 cm. Find the ratio of the area of ΔABC to that of ΔDEF.

9. The altitudes AD and PS of two similar triangles ABC and PQR are of length 2.5 cm and 3.5 cm. Find the ratio of area of ΔABC to that of ΔPQR.

10. Which of the following are right triangles?

    (i) AB = 5 cm, BC = 12 cm, CA = 13 cm

    (ii) AB = 8 cm, BC = 6 cm, CA = 10 cm

    (iii) AB = 10 cm, BC = 5 cm, CA = 6 cm

    (iv) AB = 25 cm, BC = 24 cm, 7 = 13 cm

    (v) AB = $a^2 + b^2$, BC = 2ab, CA = $a^2 - b^2$

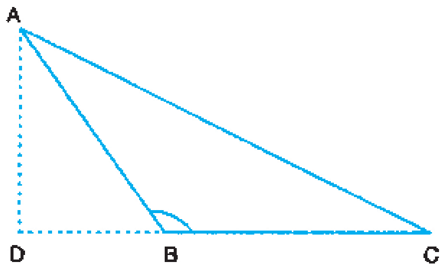


Fig. 14.39

11. Find the area of an equilateral triangle of side 2a.

12. Two poles of heights 12 m and 17 m, stand on a plane ground and the distance between their feet is 12 m. Find the distance between their tops.

13. In Fig. 13.39, show that:

    $AB^2 = AC^2 + BC^2 + 2\ BC.\ CD$

14. A ladder is placed against a wall and its top reaches a point at a height of 8 m from the ground. If the distance between the wall and foot of the ladder is 6 m, find the length of the ladder.

15. In an equilateral triangle, show that three times the square of a side equals four times the square of medians.

ANSWERS TO CHECK YOUR PROGRESS

14.1

1. (i) 6 (ii) 6 (iii) 10 cm
2. (i) No (ii) Yes (iii) Yes

14.2

1. 7.5 cm 2. 4 cm

3. $\frac{yz}{x}$ (x = –1 is not possible)

14.3

1. (i) x = 4.5, y = 3.5 (ii) x = 70, y = 50 (iii) x = 2 cm, y = 7 cm

14.4

2. 9 : 25 3. 1 : 8 5. 16 : 81 6. 4 : 5

14.5

1. (i) Yes (ii) No (iii) No (iv) Yes
2. 13 m 3. $10\sqrt{2}$ cm 8. 5 m

ANSWERS TO TERMINAL EXERCISE

3. (i) and (iii) 4. 6 cm 5. 4.5 cm 6. Yes : $\frac{AD}{DB} = \frac{AE}{EC}$

7. 4.5 cm 8. 4 : 9 9. 25 : 49 10. (i), (ii), (iv) and (v)

11. $\sqrt{3}\, a^2$ 12. 13 m 14. 10 m