Notes

18

CONSTRUCTIONS

One of the aims of studying Geometry is to acquire the skill of drawing figures accurately. You have learnt how to construct geometrical figures namely triangles, squares and circles with the help of ruler and compasses. You have constructed angles of 30^o, 60^o, 90^o, 120^o and 45^o. You have also drawn perpendicular bisector of a line segment and bisector of an angle.

In this lesson we will extend our learning to construct some other important geometrical figures.

OBJECTIVES

After studying this lesson, you will be able to

- *divide a given line segment internally in a given ratio;*
- *construct a triangle from the given data;*

  (i) SSS

  (ii) SAS

  (iii) ASA

  (iv) RHS

  (v) perimeter and base angles

  (vi) base, sum/difference of the other two sides and one base angle.

  (vii) two sides and a median corresponding to one of these sides.
- *construct a triangle, similar to a given triangle; and;*
- *Construct tangents to a circle from a point:*

  (i) on it using the centre of the circle.

  (i) outside it.

EXPECTED BACKGROUND KNOWLEDGE

We assume that the learner already knows how to use a pair of compasses and ruler to construct

- angles of 30^o, 45^o, 60^o, 90^o, 105^o, 120^o
- the right bisector of a line segment
- bisector of a given angle.

18.1 DIVISION OF A LINE SEGMENT IN THE GIVEN RATIO INTERNALLY

Construction 1: To divide a line segment internally in a given ratio.

Given a line segment AB. You are required to divide it internally in the ratio 2 : 3. We go through the following steps.

Step 1: Draw a ray AC making an acute angle with AB.

Step 2: Starting with A, mark off 5 points C_1, C_2, C_3, C_4 and C_5 on AC at equal distances from the point A.

Step 3: Join C_5 and B.

Step 4: Through C_2 (i.e. the second point), draw C_2D parallel to C_5B meeting AB in D.

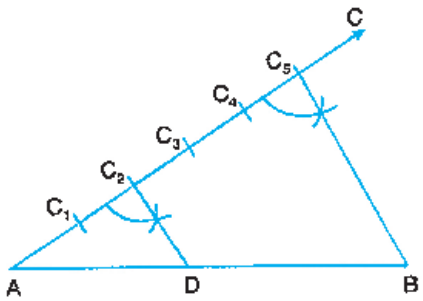

Fig. 18.1

Then D is the required point which divides AB internally in the ratio 2 : 3 as shown in Fig. 18.1.

CHECK YOUR PROGRESS 18.1

1. Draw a line segment 7 cm long. Divide it internally in the ratio 3 : 4. Measure each part. Also write the steps of construction.

2. Draw a line segment PQ = 8 cm. Find point R on it such that PR = $\frac{3}{4}$ PQ.

[**Hint:** Divide the line segment PQ internally in the ratio 3 : 1]

18.2 CONSTRUCTION OF TRIANGLES

Construction 2: To construct a triangle when three sides are given (SSS)

Suppose you are required to construct ΔABC in which AB = 6 cm, AC = 4.8 cm and BC = 5 cm.

We go through the following steps:

Step 1: Draw AB = 6 cm.

Step 2: With A as centre and radius 4.8 cm, draw an arc.

Step 3: With B as centre and radius 5 cm draw another arc intersecting the arc of Step 2 at C.

Step 4: Join AC and BC.

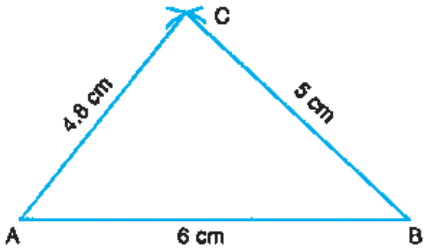

Fig. 18.2

Then ΔABC is the required triangle.

[**Note:** You may take BC or AC as a base]

Construction 3: To construct a triangle, when two sides and the included angle is given (SAS).

Suppose you are required to construct a triangle PQR in which PQ = 5.6 cm, QR = 4.5 cm and ∠PQR = 60°.

Step 1: Draw PQ = 5.6 cm

Step 2: At Q, construct an angle ∠PQX = 60°

Step 3: With Q as centre and radius 4.5 cm draw an arc cutting QX at R.

Step 4: Join PR

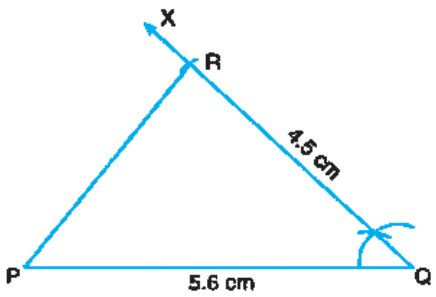

Fig. 18.3

Then ΔPQR is the required triangle.

[**Note:** You may take QR = 4.5 cm as the base instead of PQ]

Construction 4: To construct a triangle when two angles and the included side are given (ASA).

Let us construct a ΔABC in which ∠B = 60°, ∠C = 45° and BC = 4.7 cm.

To construct the triangle we go through the following steps:

Step 1: Draw BC = 4.7 cm.

Step 2: At B, construct ∠CBQ = 60°

Step 3: At C, construct ∠BCR = 45° meeting BQ at A.

Then ΔABC is the required triangle.

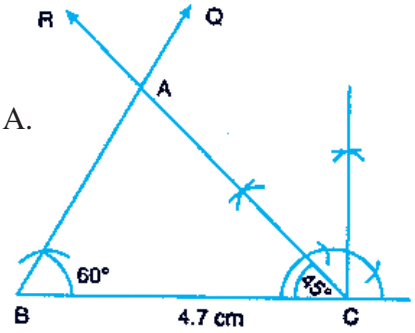

Fig. 18.4

Note: To construct a triangle when two angles and any side (other than the included side) are given, we find the third angle (using angle sum property of the triangle) and then use the above method for constructing the triangle.

Construction 5: To construct a right triangle, when its hypotenuse and a side are given.

Let us construct a right triangle ABC, right angled at B, side BC = 3 cm and hypotenuse AC = 5 cm

To construct the triangle, we go through the following steps:

Step 1: Draw BC = 3 cm

Step 2: At B, construct ∠CBP = 90°

Step 3: With C as centre and radius 5 cm draw an arc cutting BP in A.

Step 4: Join AC

ΔABC is the required triangle.

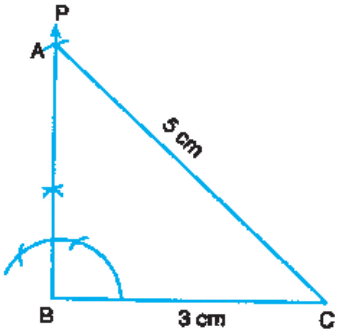

Fig. 18.5

Construction 6: To construct a triangle when its perimeter and two base angles are given.

Suppose we have to construct a triangle whose perimeter is 9.5 cm and base angles are 60° and 45°

To construct the triangle, we go through the following steps:

Step 1: Draw XY = 9.5 cm

Step 2: At X, construct ∠YXP = 30° [which is 1/2 × 60°]

Step 3: At Y, construct ∠XYQ = 22½° [which is 1/2 × 45°]

Let XP and YQ intersect A.

Step 4: Draw right bisector of XA intersecting XY at B.

Step 5: Draw right bisector of YA intersecting XY at C.

Step 6: Join AB and AC.

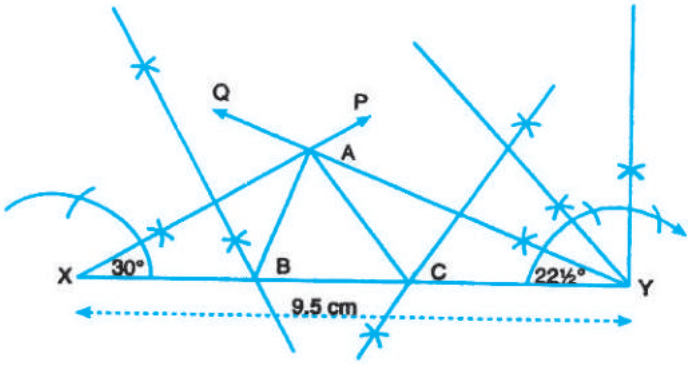

Fig. 18.6

ΔABC is the required triangle.

Construction 7: To construct a triangle when sum of two sides, third side and one of the angles on the third side are given.

Suppose you are required to construct a triangle ABC in which

AB + AC = 8.2 cm, BC = 3.6 cm and ∠B = 45°

To construct the triangle, we go through the following steps:

Step 1: Draw BC = 3.6 cm

Step 2: At B, construct ∠CBK = 45°

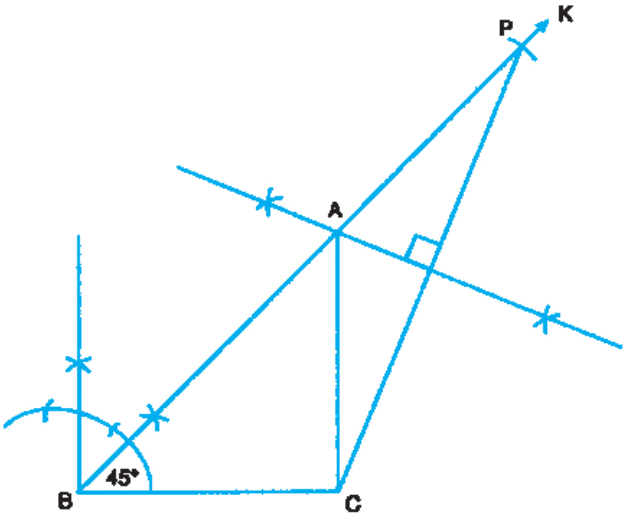

Fig. 18.7

Step 3: From BK, cut off BP = 8.2 cm.

Step 4: Join CP.

Step 5: Draw right bisector of CP intersecting BP at A.

Step 6: Join AC

ΔABC is required triangle.

Construction 8: To construct a triangle when difference of two sides, the third side and one of the angles on the third side are given.

Suppose we have to construct a ΔABC, in which BC = 4 cm, ∠B = 60°, AB – AC = 1.2 cm.

To construct the triangle we go through the following steps:

Step 1: Draw BC = 4 cm.

Step 2: Construct ∠CBP = 60°

Step 3: From BP cut off BK = 1.2 cm.

Step 4: Join CK.

Step 5: Draw right bisector of CK meeting BP produced at A.

Step 6: Join AC

ΔABC is the required triangle.

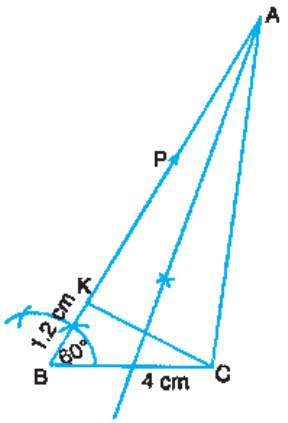

Fig. 18.8

Construction 9: To construct a triangle when its two sides and a median corresponding to one of these sides, are given:

Suppose you have to construct a ΔABC in which AB = 6 cm, BC = 4 cm and median CD = 3.5 cm.

We go through the following steps:

Step 1: Draw AB = 6 cm

Step 2: Draw right bisector of AB meeting AB in D.

Step 3: With D as centre and radius 3.5 cm draw an arc.

Step 4: With B as centre and radius 4 cm draw another arc intersecting the arc of Step 3 in C.

Step 5: Join AC and BC.

Then ΔABC is required triangle.

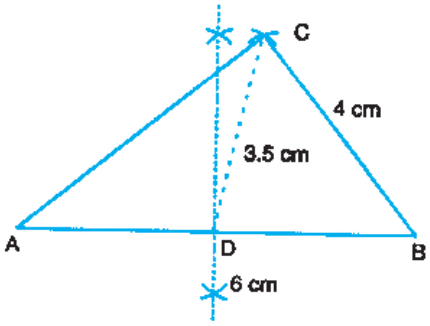

Fig. 18.9

CHECK YOUR PROGRESS 18.2

1. Construct a ΔDEF, given that DE = 5.1 cm, EF = 4 cm and DF = 5.6 cm. Write the steps of construction.

   Note: *You are also required to write the steps of construction in each of the remaining problems.*

2. Construct a ΔPQR, given that PR = 6.5 cm, ∠P = 120° and PQ = 5.2 cm.
3. Construct a ΔABC given that BC = 5.5 cm, ∠B = 75° and ∠C = 45°.
4. Construct a right triangle in which one side is 3 cm and hypotenuse is 7.5 cm.
5. Construct a right angled isoceles triangle in which one of equal sides is 4.8 cm.
6. Construct a ΔABC given that AB + BC + AC = 10 cm, ∠B = 60°, ∠C = 30°.
7. Construct a ΔABC in which AB = 5 cm, ∠A = 60°, BC + AC = 9.8 cm.
8. Construct a ΔLMN, when ∠M = 30°, MN = 5 cm and LM – LN = 1.5 cm.
9. Construct a triangle PQR in which PQ = 5 cm, QR = 4.2 cm and median RS = 3.8 cm.

18.3 TO CONSTRUCT A TRIANGLE SIMILAR TO A GIVEN TRIANGLE, AS PER GIVEN SCALE FACTOR

[Here, **Scale Factor** means the ratio of the sides of the triangle to be constructed, to the corresponding sides of the given triangle.]

Construction 10: Construct a triangle similar to a given triangle ABC with its sides equal to 3/5 of the corresponding sides of the triangle ABC.

Steps of Construction:

1. Let ABC be the given Δ. Draw any ray BX making an acute angle with BC on the side opposite to vertex A.
2. Locate 5 points B_1, B_2, B_3, B_4 and B_5 on BX so that

   $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$
3. Join B_5C and draw a line through B_3 parallel to B_5C to meet BC at C′.
4. Draw a line though C′ parallel to CA to meet AB in A′.

   Then ΔA′BC′ is the required Triangle.

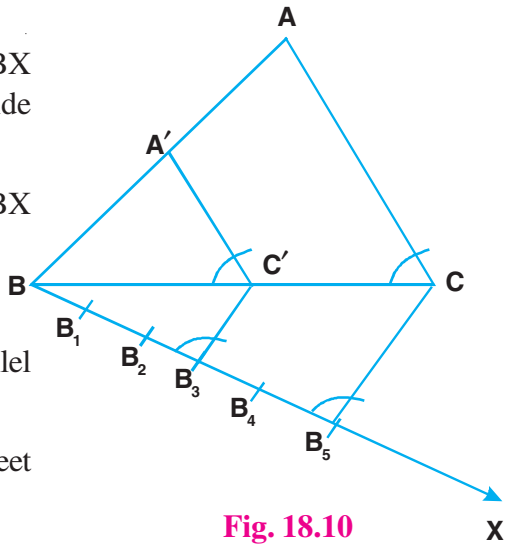

Fig. 18.10

Construction 11: Construct a triangle with sides 5cm, 6 cm and 7 cm. Construct another triangle similar to this triangle with scale factor $\frac{2}{3}$.

Steps of Construction:

1. Draw of a line segment BC = 7 cm
2. Through B draw an arc of radius 6 cm. Through C draw another arc of radius 5 cm to intersect the first arc at A.
3. Join AB and AC to get ΔABC.
4. Draw a ray BX making an acute angle with BC.
5. Locate 3 points B_1, B_2 and B_3 on BX such that $BB_1 = B_1B_2 = B_2B_3$
6. Join B_3C and through B_2 draw a line parallel to B_3C to meet BC in C′.
7. Through C′, draw a line parallel to CA to meet AB at A′.

   Then A′BC′ is the required triangle.

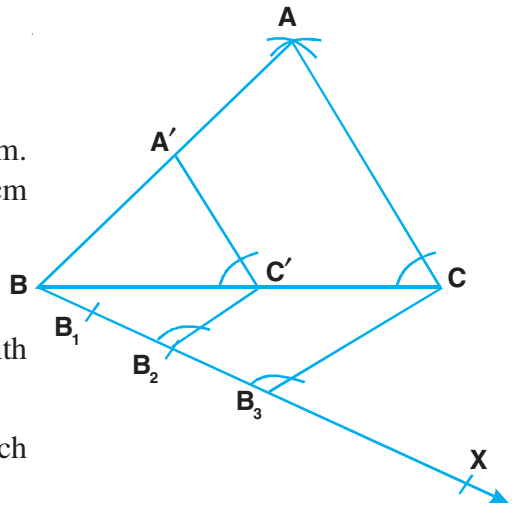

Fig. 18.11

CHECK YOUR PROGRESS 18.3

1. Construct a triangle of sides 4cm, 5 cm and 7 cm and then a triangle similar to it whose sides are $\frac{3}{4}$ of the corresponding sides of the first triangle.
2. Draw a triangle ABC with BC = 7 cm, AB = 5 cm and ∠ABC = 60°. Then construct a triangle whose sides are $\frac{4}{5}$ of the corresponding sides of the triangle ABC.
3. Draw a right triangle with sides (other than hypotenuse) of lenghts 5 cm and 6 cm. Then construct another triangle similar to this triangle with scale factor $\frac{4}{5}$.
4. Draw a ΔABC with base BC = 6 cm, ∠ABC = 60° and side AB = 4.5 cm. Construct a triangle A′BC′ similar to ABC with scale factor $\frac{5}{6}$.

18.4 CONSTRUCTION OF TANGENTS TO A CIRCLE

Construction 12: To draw a tangent to a given circle at a given point on it using the centre of the circle.

Suppose C be the given circle with centre O and a point P on it. You to draw a tangent to the circle. We go through the following steps:

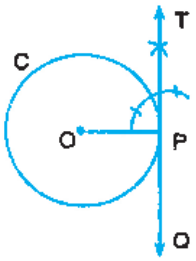

Fig. 18.12

Step 1: Join OP.

Step 2: At P, draw PT ⊥ OP.

Step 3: Produce TP to Q

Then TPQ is the required tangent.

Construction 13: To draw tangents to a circle from a given point outside it.

Suppose C be the given circle with centre O and a point A outside it. You have to draw tangents to the circle from the point A. For that, we go through the following steps:

Step 1: Join OA.

Step 2: Draw the right bisector of OA. Let R be mid point of OA.

Step 3: With R as centre and radius equal to RO, draw a circle intersecting the given circle at P and Q.

Step 4: Join AP and AQ.

Then AP and AQ are the two required tangents.

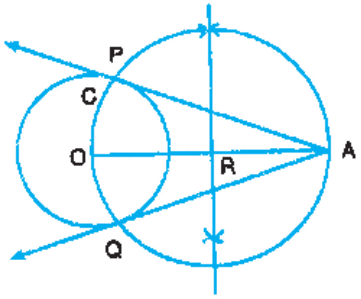

Fig. 18.13

CHECK YOUR PROGRESS 18.4

1. Draw a circle of 3 cm radius. Take a point A on the circle. At A, draw a tangent to the circle by using the centre of the circle. Also write steps of construction.
2. Draw a circle of radius 2.5 cm. From a point P outside the circle, draw two tangents PQ and PR to the circle. Verify that lengths of PQ and PR are equal. Also write steps of construction.

TERMINAL EXERCISE

1. Draw a line segment PQ = 8 cm long. Divide it internally in the ratio 3 : 5. Also write the steps of construction.

   Note: *You are also required to write the steps of construction in each of the following problems.*

2. Draw a line segment AB = 6 cm. Find a point C on AB such that AC : CB = 3 : 2. Measure AC and CB

3. Construct a triangle with perimeter 14 cm and base angles 60^o and 90^o.
4. Construct a right angled triangle whose hypotenuse is 8 cm and one of its other two sides is 5.5 cm.
5. Construct a ΔABC in which BC = 3.5 cm, AB + AC = 8 cm and $\angle B = 60^o$.
6. Construct a ΔABC in which AB = 4 cm, $\angle A = 45^o$, and AC – BC = 1 cm.
7. Construct a ΔPQR with PQ = 5 cm, PR = 5.5 cm and the base QR = 6.5 cm. Construct another triangle P′QR′ similar to ΔPQR such that each of its sides are $\frac{5}{7}$ times the corresponding sides of ΔPQR.
8. Construct a right triangle with sides 5 cm, 12 cm and 13 cm. Construct another triangle similar to it with scale factor 5/6.
9. Draw a circle of diameter 6 cm. From a point P outside the circle at a distance of 6 cm from the centre, draw two tangents to the circle.
10. Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle.