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11

GRAVITATION

In previous chapter you have learnt that a force is required to change the state of rest or of motion of a body. You are also aware that all objects when dropped from a height fall towards the earth. Why do objects fall towards the earth? You might think that this must be due to some force known as force due to gravity or gravitational force. In this lesson we will learn about gravitation, force of gravity and motion of bodies under the influence of gravity.

We shall also discuss about buoyancy and Archimedes' principle.

OBJECTIVES

After completing this lesson you will be able to:

- *illustrate the existence of force of gravitation;*
- *state Newton's law of gravitation;*
- *explain the term acceleration due to gravity;*
- *modify equations of motion of an object falling under gravity;*
- *solve problems relating to one dimensional motion under gravity;*
- *distinguish between mass and weight and find the relation between them;*
- *define free fall motion and explain weightlessness;*
- *illustrate the force of buoyancy experienced by a body immersed wholly or partly in a fluid and*
- *state the principle of Archimedes and apply it to solve problems.*

11.1 FORCE OF GRAVITATION

It is our everyday experience that bodies thrown vertically upward come back to the earth. Even if an object is dropped from some height, it falls towards the earth. Similarly tree leaves and fruits fall toward the earth when they are separated from the branches. Why does it happen so? This must be due to some force acting on the bodies like leaves or fruits. What type of force is acting on them? It was Issac Newton who answered this question.

There is an interesting story about Newton. It is said that while Newton was sitting under an apple tree, an apple fell on him. The fall of the apple set Newton thinking, why did the apple fall down? If some force is acting on the apple then it must be in accelerated motion. Let us try to understand this with the help of an activity.

ACTIVITY 11.1

Release a small stone from your hand from a height of about 1 metre. Observe its speed just before it hits the ground. Now, release the same stone from a height of about 5 metres (say from first floor of the house) (Fig. 11.1). Again observe its speed just before it hits the ground. Ensure that in each case the stone is released without pushing. Did the stone possess the same speed just before it hits the ground in both the cases? In which case the stone strike the ground faster? Can you identify the force which accelerated the stone?

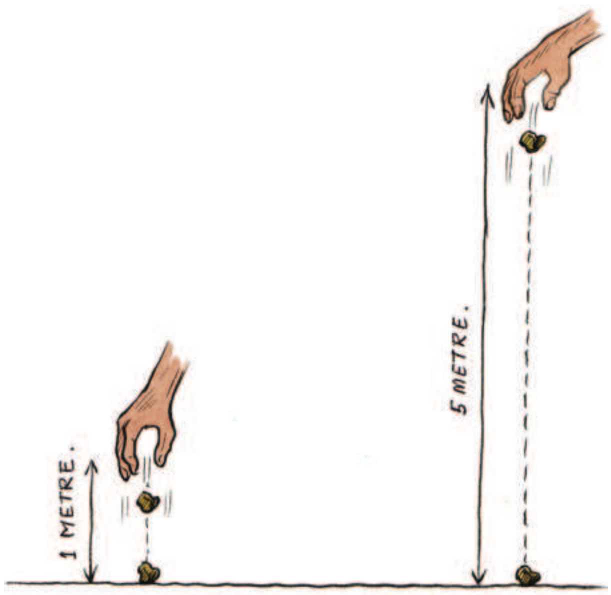


Fig. 11.1 *A stone falling from different heights*

In above activity you have observed that the force of attraction due to earth accelerated the stone. Newton knew that bodies fall towards the earth due to force of gravity. He further thought, if the earth can attract an apple or a stone, can it also attract the moon? He was also curious to know whether the same force was responsible for keeping the planets go around the sun in their orbits.

Newton concluded that in order to move in a circular orbit the moon must be attracted by the earth continuously. Arguing in the same lines he said that there exists a force between the sun and the planets. The force is known as the gravitational force. He stated that gravitational force exists everywhere in the universe. All objects in the universe attract each other. The interesting aspect of the gravitational force is that it is always attractive whatever may be the size of bodies.

11.2 NEWTON'S LAW OF GRAVITATION

On the basis of his observations, Newton expressed the law of gravitation in the language of mathematics. He stated the law as follows:

Every particle in the universe attracts every other particle with a force. This force is proportional to the product of their masses and inversely proportional to the square of the distance between them. The force is along the line joining the two particles. Mathematically,

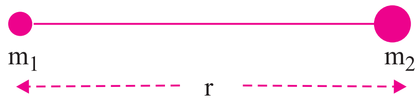


Fig. 11.2 *Newton's law of gravitation*

$$F \propto \frac{m_1 m_2}{r^2}$$

where m_1 and m_2 are the masses of the two particles separated by a distance r.

or $$F = G\frac{m_1 m_2}{r^2} \qquad ...(11.1)$$

where G is a constant of proportionality. It is called the universal gravitational constant. Its value is same everywhere on the earth or in the universe.

In SI units, where m is measured in kilogram, F in newton, r in metre, the accepted value of G is 6.67×10^{-11} Nm^2kg^{-2}. As the value of G is very small, you can realize that the force of gravitation between objects of ordinary mass is very weak.

Let us find out how much the force of attraction between you and your friend sitting on the next bench at a distance of 1 metre apart is. If you are of say 50 kg and your friend is of 40 kg then the force of attraction would be,

$$F = \frac{6.67 \times 10^{-11} \times 40 \times 40}{1 \times 1}$$

$$= 13340 \times 10^{-11} \text{ N}$$

$$= 113.34 \times 10^{-8} \text{ N}$$

You will appreciate that this force is very weak. It is at least a hundred times weaker than the force exerted by a small piece of paper on the pan of a balance. You can also realize how weak the force of gravitation is, when you lift a small stone or when a charged comb picks up small pieces of paper. However, the force of gravitation becomes appreciably stronger if masses of the objects are increased.

Example 11.1: A boy of 40 kg mass is standing on the surface of earth. If the mass of the earth is 6×10^{24} kg and its radius is 6.37×10^{6} m, then find the force of attraction between the boy and the earth. Take the value of G as 6.67×10^{-11} Nm^2 kg^{-2}.

Solution: Mass of the earth = 6×10^{24} kg

Mass of the boy = 40 kg

Radius of the earth = 6.37×10^{6} m

(This is the distance separating the boy from the centre of the earth)

Value of $G = 6.67 \times 10^{-11}$ Nm^{2}kg^{-2}

The force of attraction (F) between the boy and the earth

$$= \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 40}{6.37 \times 10^{6} \times 6.37 \times 10^{6}} = 394.5 \text{ N}$$

Now you can appreciate that the force with which the earth and the boy attract each other is more than a thousand million times stronger than the force of attraction between you and your friend sitting at a distance of about 1 metre from you.

The gravitational force due to earth is also known as **gravity**. Thus, when we are dealing with very large masses like the earth, the moon or the sun, the gravitational force between such objects is quite large.

INTEXT QUESTIONS 11.1

1. Why do two students sitting close to each other not feel force of gravitational attraction between them?
2. Distance between two bodies is increased by a factor of four. How much will be the change in the force of gravitation?
3. Why is G known as universal gravitational constant?

11.3 ACCELERATION DUE TO GRAVITY

In activity 11.1 we have seen that the speed of a falling stone increases continuously. From this activity we concluded that the stone was accelerated due to force of attraction between the stone and the earth. Can we give some special name to this

Notes

acceleration? This acceleration is called the acceleration due to gravity. Is this acceleration large if the stone has a large mass? Do heavier objects fall faster than lighter one? Let us find out.

ACTIVITY 11.2

Caution: while performing this activity, be careful not to hurt anyone.

Ask one of your friends to stand at the roof top of a two storied building with stones of different masses in his two hands (Fig. 11.3). Ask him to drop these stones together. Carefully observe falling of the stones. What do you find? Why both the stones reach the ground at the same time?

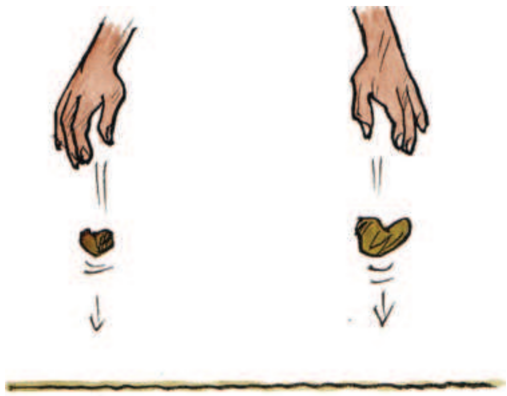
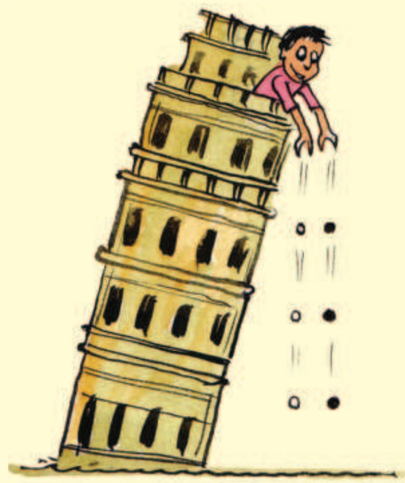


Fig. 11.3 *Dropping two stones of different masses together.*

Do you know

According to a story, Galileo dropped different objects from the leaning tower of Pisa in Italy to prove that objects of different masses fall at the same rate.



You can perform the above activity in an interesting manner.

ACTIVITY 11.3

Drop a five rupee coin and a paper (15 cm × 15 cm) simultaneously from the same height. What do you observe? You will find that the coin falls to the ground much before the paper does. What do you conclude from this observation? You may be tempted to conclude that the heavier objects fall faster than the higher ones.

Now crumple the paper into a small ball. Again drop the coin and the crumpled paper ball simultaneously from the same height. What do you observe now? You will find that both the coin and the paper ball hit the ground at the same time. In the first case the slowing down of paper was due to friction offered by air. Large surface encounters more resistance by air. What conclusion can be drawn from this activity?

This activity shows that two objects of different masses would reach the ground together when dropped from the same height. Think why?

Do you know

A British scientist Robert Boyle placed a coin and feather in a glass tube. He used a vacuum pump to remove air from the glass tube. When the tube was inverted both the coin and the feather hit the bottom at the same time.

The earth's gravity accelerates the coin and paper ball in the downward direction. Since both the coin and paper ball reach the ground together, this acceleration called acceleration due to gravity (g), is same for both of them. Infact, acceleration due to gravity is same for any mass at a given place. The SI unit of g is same as that of acceleration, i.e., ms^{-2}.

Let us try to find out an expression for the acceleration due to gravity. Let the mass of the stone falling from a height (in activity 11.1) be m. The acceleration involved in falling stone due to earth's gravity is denoted by 'g'.

We know that force is product of mass and acceleration. Therefore, the magnitude of force of gravity 'F' will be equal to product of mass and acceleration due to gravity.

$$F = mg \qquad (11.2)$$

From equations (11.1) and (11.2), we have

$$mg = G\frac{Mm}{r^2}$$

or $$g = \frac{GM}{r^2} \qquad ...(11.3)$$

where M is the mass of the earth and r is the distance between the object and the centre of the earth. If the object is on or near the surface of the earth, the distance r in equation (11.3) will be equal to the radius of the earth R. Thus,

$$g = G\frac{M}{R^2} \qquad ...(11.4)$$

Thus we see that the value of 'g' is independent of the mass of the freely falling body. The radius of the earth is not same at all the places on the surface of the earth. So the value of 'g' changes from place to place on the earth. Its value is greater at the poles than at the equator. The average value of 'g' on and near the surface of the earth is taken as 9.8 ms^{-2}.

11.4 MOTION OF AN OBJECT UNDER GRAVITY

We know that g is constant near the surface of earth. Therefore, all the equations for uniformly accelerated motion of bodies (discussed in Chapter 9) become valid when acceleration a is replaced by g. Can you write now the modified equations of motion? These are:

$$v = u + gt \qquad ...(11.5)$$

$$s = ut + \frac{1}{2}gt^2 \qquad ...(11.6)$$

$$v^2 = u^2 + 2gs \qquad ...(11.7)$$

where u and v are the initial and final velocities and s is the distance covered in time t.

Example 11.2: Take the mass of the earth to be 6 $\times 10^{24}$ kg and its radius as 6.4 $\times$ 10^6 m. Calculate the value of g. (G = 6.7 $\times$ 10^{-11} N m^2 kg^{-2}).

Solution: From equation 11.4,

$$g = G\frac{M}{R^2}$$

$$= \frac{6.7\times10^{-11}\,\text{Nm}^2\text{kg}^{-2}\times6\times10^{24}\,\text{kg}}{\left(6.4\times10^6\,\text{m}\right)^2}$$

$$= 9.8\ \text{ms}^{-2}$$

Example 11.3: The mass of the earth is 6 $\times 10^{24}$ kg and that of the moon is 7.4 $\times$ 10^{22} kg. If the distance between the earth and the moon is 3.84 $\times$ 10^8 m, calculate the force exerted by the earth on the moon. G = 6.7 $\times$ 10^{-11} N m^2 kg^{-2}.

Solution. The mass of the earth, m_1 = 6 $\times$ 10^{24} kg

The mass of the moon, m_2 = 7.4 $\times 10^{22}$ kg

The distance between the earth and the moon, r = 3.84 $\times$ 10^8 m

$$G = 6.7 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$$

From equation (11.1) the force exerted by the earth on the moon is

$$F = G\frac{m_1 m_2}{r^2}$$

$$= \frac{6.7\times10^{-11}\,\text{Nm}^2\text{kg}^{-2}\times6\times10^{24}\,\text{kg}\times7.4\times10^{22}\,\text{kg}}{\left(3.84\times10^8\,\text{m}\right)^2}$$

$$= 2.01 \times 10^{20}\ \text{N}$$

Example 11.4: A ball is thrown vertically upwards and rises to a height of 122.5 m. Calculate

(i) the velocity with which the ball was thrown upwards and

(ii) the time taken by the ball to reach the highest point.

(Take g = 9.8 ms^{-2})

Solution: Distance travelled, s = 122.5 m

Final velocity, v = 0 ms^{-1}

Acceleration due to gravity, g = 9.8 ms^{-2}

(i) From equation (11.7) $\quad v^2 = u^2 = 2gs$

$$0 = u^2 + 2(-9.8\ \text{ms}^{-2}) \times 122.5\ \text{m}$$

For upward motion g is taken as negative.

$$\therefore \quad -u^2 = -2 \times 9.8 \times 122.5\ \text{m}^2\text{s}^{-2}$$

$$u^2 = 2401\ \text{m}^2\text{s}^{-2}$$

$$u = 49\ \text{ms}^{-1}$$

Thus the velocity with which the ball was thrown upwards is 49 ms^{-1}.

(ii) From equation (11.5), $\quad v = u + gt$

$$0 = 49\ \text{ms}^{-1} + (9.8\ \text{ms}^{-2}) \times t$$

Therefore, $\quad t = \dfrac{49}{9.8}\text{s} = 5\text{s}$

Thus,

(i) Initial velocity = 49 ms^{-1}; and

(ii) Time taken = 5s

INTEXT QUESTIONS 11.2

1. What do you mean by acceleration due to gravity?
2. Why do a heavier and a lighter object when dropped from a same height fall at the same rate?
3. State SI unit of acceleration due to gravity.
4. Write equations of motion of an object moving under gravity.

11.5 MASS AND WEIGHT

11.5.1 Mass

Mass of a body is the quantity of matter contained in the body. Mass of an object is constant and does not change from place to place. It remains the same whether the object is on earth, on moon or anywhere in outer space. The mass of an object is measured with the help of a pan balance.

We have also learnt in previous chapter that mass of an object is the measure of its inertia. It means that greater the mass, the greater is the inertia of the object.

11.5.2 Weight

The weight of an object is the force with which it is attracted towards the earth. Can you recall the relation between force and acceleration?

$$\text{Force} = \text{Mass} \times \text{Acceleration}$$

Therefore,
$$F = mg \quad (11.8)$$

If weight of an object is denoted by W, then

$$W = mg \quad (11.9)$$

As weight is a force, therefore, its SI unit is the same as that of the force. Try to recall this unit. It is newton. Its symbol is N. This force (weight) acts vertically downwards. It has both magnitude and direction. The weight of an object is generally measured by a spring balance.

From equation (11.9) we see that weight of an object depends on its mass and value of g. As the value of g is constant at a given place, therefore, the weight of the object at a given place is directly proportional to its mass. However, the weight of an object will be different on different parts of the earth as the value of g is different on different parts of the earth.

11.5.3 Weightlessness

You may have noticed increase in weight while in moving in Lift/Elevator upward and decrease in weight when moving downward. Similar case you can experience in merry-go-round. Also you have heard that an astronaut experiences weightlessness in space. What does the term weightlessness mean?

ACTIVITY 11.4

Hold a heavy book in your hand as shown in Fig. 11.4. Can you feel the weight of the book on your hand? Now move your hand quickly in the downward direction with some acceleration. What do you feel? Do you feel some decrease in the weight of the book? Can you explain the reason for this decrease in the weight?

Book in the hand *Hand moving downward*

Fig. 11.4

We usually measure the weight by a spring balance or a weighing machine which rests on a rigid floor. How does a weighing machine record the weight of an object?

Suppose a child is standing on a weighing machine which rests on the floor. The child exerts a downward force equal to his weight W on the machine (Fig. 11.5).

Fig. 11.5 *A child on a weighing machine*

According to the third law of motion the machine exerts an upward reaction 'R' on the boy which is equal to W. The weighing machine measures the reaction R, which is the weight of the boy.

Now imagine that the floor below the weighing machine is suddenly removed. What would happen? The boy and the scale would fall towards the earth with the same acceleration. In this case the boy cannot exert a force on the weighing machine. The weighing machine in this case would show a zero weight. Thus we can conclude that a body falling freely under gravity is weightless.

Now you can understand why an astronaut experiences weightlessness in a spaceship. The spaceship with the astronaut falls freely towards the earth. The astronaut therefore, appears to be floating weightlessly (Fig. 11.6).

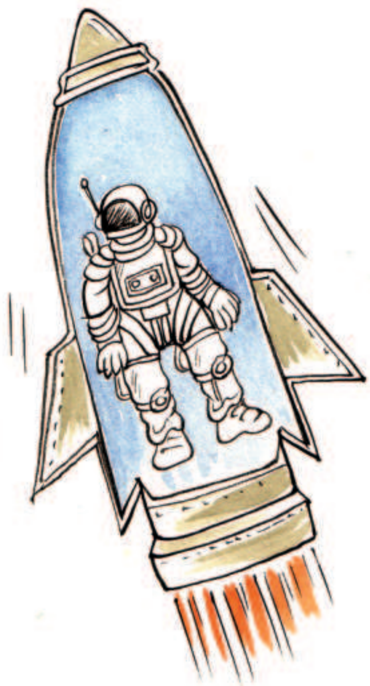


Fig. 11.6 *An astronaut in a spaceship*

INTEXT QUESTIONS 11.3

1. Write two differences between mass of an object and its weight.
2. State two factors on which weight of an object depends.
3. What will be the weight of an apple while it is falling from a tree?

11.6 BUOYANCY AND ARCHIMEDES' PRINCIPLE

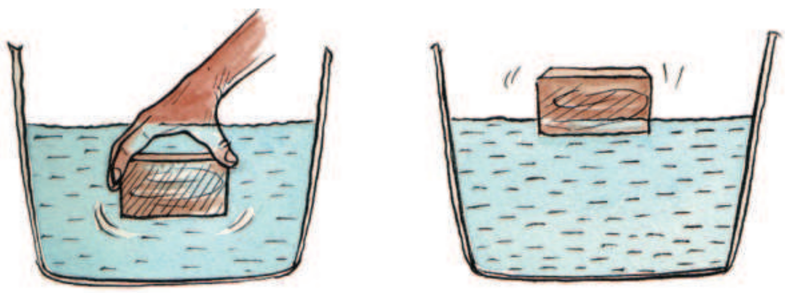
11.6.1 Buoyancy

Have you ever experienced that a mug filled with water appears to be heavier when it is lifted from bottom of the bucket to above the surface of water than the mug within the water in the bucket. Why is it so? Let us understand it with the help of an activity.

ACTIVITY 11.5

Take a large wooden block and put it in a bucket filled with water. What do you observe? You will see that the wooden block floats when placed on the surface of water.

Now push the block into the water. What do you feel? Why do you feel an upward push on your hand? What does it indicate? This indicates that water exerts an upward force on the wooden block. Now, push the wooden block further down till it is completely immersed in water [Fig. 11.7(a)]. Release the wooden block. What do you observe? The block bounces back to the surface of water [Fig. 11.7(b)].



(a) *(b)*

Fig. 11.7 *(a) Wooden block immersed in water (b) The block becomes back when released*

The upward force exerted by the water on the wooden block is known as the **force of buoyancy** or **buoyant force**. This force is also known as **upthrust**. In fact, all bodies experience a buoyant force when they are immersed in a fluid that is a liquid or a gas. Can you cite some more examples of buoyant force?

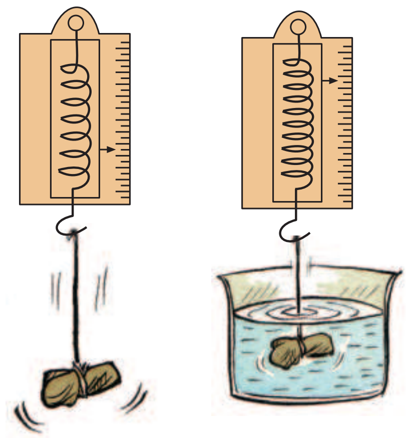
What is the magnitude of the buoyant force experienced by an object? Do all objects in a given fluid experience the same buoyant force? Is not same for all fluids for a given object? You can answer all such questions after studying Archimedes' principle.

11.6.2 Archimedes' Principle

ACTIVITY 11.6

Take a piece of stone and suspend it from a spring balance with the help of a thread [Fig. 11.8(a)]. Note the reading of the spring balance. This is the weight of the stone in air. Now, dip the stone slowly in to water kept in a container [Fig. 11.8(b)]. Observe carefully. What happens to the reading on the balance?

You will find that the reading of the spring balance decreases as the stone is gradually lowered in water. However, when the stone gets fully immersed in water, no further change is observed in the reading of the spring balance. What do you infer from this observation? Decrease in the reading of the spring balance shows that an upward force acts on the stone when it is dipped in water. As discussed earlier this upward force is known as the force of buoyancy. Archimedes discovered a principle to determine the magnitude of the force of buoyancy.



(a) *(b)*

Fig. 11.8 *Reading of the spring balance decreases when the stone is immersed in water*

Archimedes' principle is stated as follows:

When a body is immersed fully or partially in a fluid, it experiences an upward force that is equal to the weight of the fluid displaced by it.

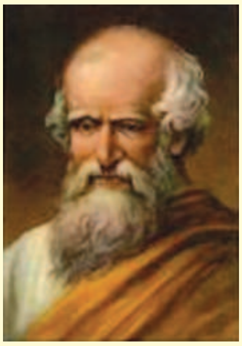
From Archimedes' principle it is clear that the magnitude of the buoyant force acting on a body at a given place depends on

- density of the fluid and
- volume of the body immersed in the fluid.

Archimedes' principle has many applications. It is used in designing ships and submarines. Hydrometers which are used to determine the density of liquids are based on Archimedes's principle. Lactometers, which are used for determining the purity of milk, are also based on this principle.

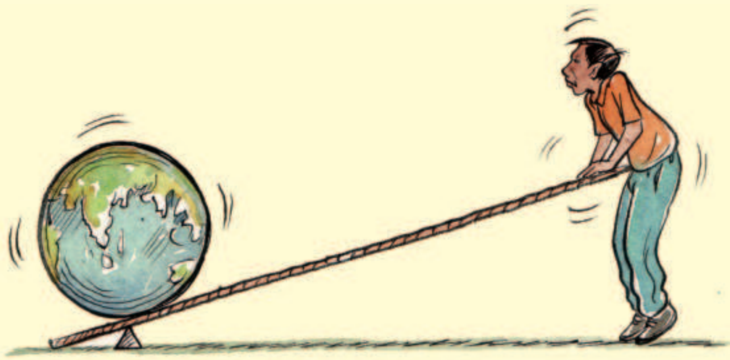
Do you know

Archimedes was a great Greek mathematician and scientist. He is best known for his famous Archimedes' principle. It is said that Archimedes discovered this principle when he stepped in a bathtub full of water and noticed that water overflowed from it. He ran through the streets shouting "Eureka", "Eureka", ..., which means, "I found it".



Archimedes
(287 BC-212 BC)

He invented the famous Archimedes's screw which was used for raising water from a lower to a higher level. His work in the field of mechanics and geometry made him famous. About levers once he said that, "give me a bar, long and strong enough, and a place to stand and I will lift the Earth."



INTEXT QUESTIONS 11.4

1. Hold a mug full of water inside a bucket filled with water. Now lift it above the surface of water. Why do you feel it is heavier now?
2. Why does a piece of cork released under water bounce back?
3. What do you mean by buoyant force?
4. Does the buoyant force act on a body when it is kept in vacuum?
5. State two applications of Archimedes' principle.

WHAT YOU HAVE LEARNT

- Newton's law of gravitation states that every particle in the universe attracts every other particle with a force, which is proportional to the product of their masses and inversely proportional to the square of the distance between them.
- Force of gravitation between objects of ordinary mass is very weak. However, when large masses are involved this force becomes appreciably stronger.
- The gravitational force due to earth is known as gravity.
- The value of acceleration due to gravity is independent of the mass of the body.
- The weight of an object is the force with which it is attracted towards the earth. It is equal to the product of mass and acceleration due to gravity.
- The mass of an object is constant and does not vary from place to place. However the weight of an object may vary from place to place.
- A body falling freely under gravity is weightless.
- All objects experience a buoyant force when they are immersed in a fluid.
- The magnitude of the buoyant force acting on a body at a given place depends on density of the fluid and volume of the body immersed in the fluid.
- Archimedes' principle states that when a body is immersed fully or partially in a fluid, it experiences an upward force that is equal to the weight of the fluid displaced by it.

TERMINAL EXERCISE

1. State Newton's law of gravitation.
2. How does the force of gravitation between two objects change when the distance between them is doubled?
3. How does the gravitational force between two objects change if the masses of both objects are doubled?
4. Derive an expression for the acceleration due to gravity on the surface of the earth in terms of earth's mass, gravitational constant and radius of earth.
5. Write the equations of motion of an object moving or falling only under gravity.
6. What are the differences between the mass of an object and its weight? On what factors does the weight of an object depend?
7. Why does a capped empty plastic bottle released under water bounces back to the surface of water?

8. What is force of buoyancy? What are the factors on which the magnitude of the buoyant force acting on a body at a given place depends?
9. State Archimedes' principle. Give two applications of Archimedes' principle.
10. If the average distance between the earth and the sun is 1.5×10^{11} m, calculate the force of gravitation between the two. Given:

$$\text{mass of the earth} = 6 \times 10^{24}\ \text{kg}$$
$$\text{mass of the sun} = 2 \times 10^{30}\ \text{kg}$$
$$\text{value of } G = 6.7 \times 10^{-11}\ \text{Nm}^2\text{kg}^{-2}$$

11. What is the mass of an object whose weight is 49N? (Given $g = 9.8\ \text{ms}^{-2}$).
12. A stone is dropped from the top of a tower 45 m high. What is its velocity when it hits the ground? (Given $g = 10\ \text{ms}^{-2}$).
13. A body weighs 3.5 N in air and 2 N in water. How much buoyant force acts on the body?
14. A body is immersed in a liquid. If the liquid displaced by the body weighs 1 N then what is the buoyant force acting on the body?

ANSWERS TO INTEXT QUESTIONS

11.1

1. Gravitational force is extremely weak. Therefore, small masses do not attract each other due to this force.
2. As the force of gravitation is inversely proportional to the square of the distance between two bodies, the force will decrease by a factor of 1/16.
3. The value of G is same everywhere on the earth or in the universe. Therefore, G is known as universal gravitational constant.

11.2

1. The acceleration produced due to force of attraction by the earth is known as acceleration due to gravity.
2. Because the acceleration due to gravity is same for both heavy and light objects.
3. SI unit for acceleration due to gravity is ms^{-2}.
4. Equations of motion

$$v = u + gt \qquad ...(1)$$

$$s = ut + \frac{1}{2}gt^2 \qquad ...(2)$$

$$v^2 = u^2 + 2gs \qquad ...(3)$$

11.3

1. Mass is the quantity of matter contained in a body.

   Man of a body remains the same at all places.

   Weight of an object on earth is the force with which it is attracted towards the earth. Weight of an object changes from place to place.
2. Weight of an object depends upon
   - (i) mass of the body
   - (ii) acceleration due to gravity.
3. Zero

11.4

1. When immersed in water a buoyant force acts on the mug. Therefore, it feels lighter inside water. When lifted above the surface of water it feels heavier.
2. Due to buoyant force (or upthrust)
3. When an object is immersed in a fluid it experiences an upward force which is known as buoyant force.
4. No
5. Applications of Archimedes's principle:
   - (i) In designing ships and submarines
   - (ii) Hydrometers or lactometers

MODULE - 4

ENERGY

12. Sources of Energy
13. Work and Energy
14. Thermal Energy
15. Light Energy
16. Electrical Energy
17. Magnetic Effect of Electric Current
18. Sound and Communication